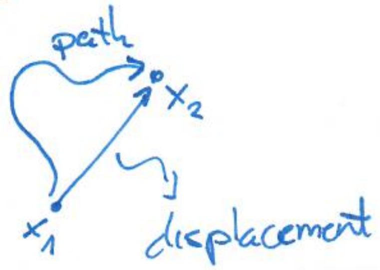
KINEMATICS



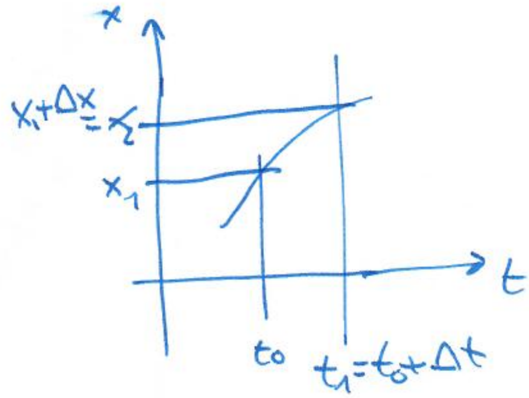
$l \equiv$ distance = length of traveled path

displacement $\neq l$

$$\Delta \bar{x} = \bar{x}_2 - \bar{x}_1$$

Average velocity.

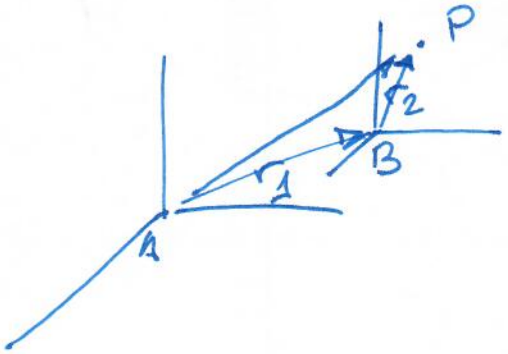
$$\bar{v}_m = \frac{\Delta \bar{x}}{\Delta t} = \frac{\text{distance}}{\text{time}}$$



Instantaneous velocity.-

$$\vec{v}_i = \lim_{\Delta t \to 0} \frac{\Delta \bar{x}}{\Delta t} = \frac{d\bar{x}}{dt}$$

Motions and reference frames



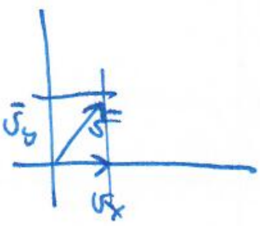
$\vec{r}_{AP} = \bar{r}_1 + \bar{r}_2$ positions

$\bar{v}_{AP} = \bar{v}_1 + \vec{v}_2$ velocities

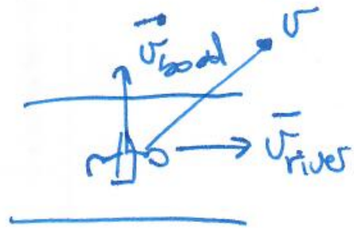
Composition of motions

* Projectile

$\bar{v} = \bar{v}_x + v_y$



* River



$\vec{v} = \bar{v}_{river} + \bar{v}_{boat}$

Acceleration

$$\bar{a}_m = \frac{d\bar{v}}{dt}$$

$$\bar{a}_i = \frac{d\bar{v}}{dt} = \frac{d^2\bar{x}}{dt^2}$$

Example

$$\bar{a} = \text{constant}$$

$$dv = a\,dt \; ; \quad v - v_0 = a(t - \overset{0}{t_0})$$

$$\boxed{v = v_0 + at}$$

$$dx = v\,dt \Rightarrow x - x_0 = \int v\,dt$$

$$x - x_0 = \int (v_0 + at)\,dt$$

$$\boxed{x - x_0 = v_0 t + \frac{1}{2} a t^2}$$

Moreover: $a = \frac{v - v_0}{t}$

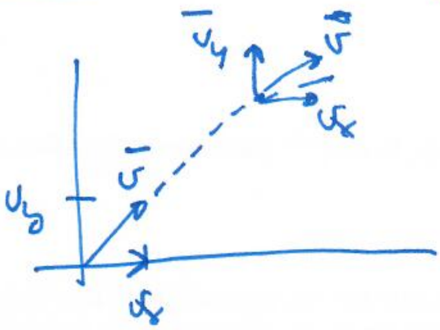
$$x - x_0 = \frac{1}{2}(v - v_0)t$$

$$\frac{\Delta x}{t} = \underset{\text{speed}}{\text{average}} = \frac{1}{2}(v + v_0)$$

Proto-Typical motion - example

$\bar{a} = \text{constant} \Rightarrow \vec{a} = \vec{g}$ free fall.

Kinematics of projectile motion



$$\vec{v} = \vec{v}_x + \vec{v}_y$$

$$\begin{cases} v_x = v_0 \cos\theta \\ v_{0y} = v_0 \sin\theta \end{cases}$$

$$\begin{cases} v_x = v_{0x} & \text{constant} \\ v_y = v_{0y} - gt \end{cases}$$

$$x(t) = x = x_0 + v_{0x} t$$

$$y(t) = y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$\boxed{y(x) = (\operatorname{tg} \theta_0) x - \left(\frac{g}{2 v_0^2 \cos^2\theta_0}\right) x^2} \quad \text{parabola}$$

Maximum range

$$y(t) = 0 \quad \leftarrow \text{condition}$$

$$\begin{cases} t_1 = 0 \\ t_2 = \dfrac{2 v_{0y}}{g} = \text{time-of-flight} = t_{max} \end{cases}$$

the horizontal displacement corresponding to t_{max}

$$x(t_{max}) = x_0 + v_{0x} \frac{2 v_{0y}}{g} = x_0 + v_0 \frac{\cos\theta \, 2 v_0 \sin\theta}{g}$$

as $2 \sin\theta \cos\theta = \sin(2\theta)$

then $$\boxed{x(t_{max}) = \frac{v_0^2}{g} \sin(2\theta)}$$

This maximum range can be seen as a function on θ

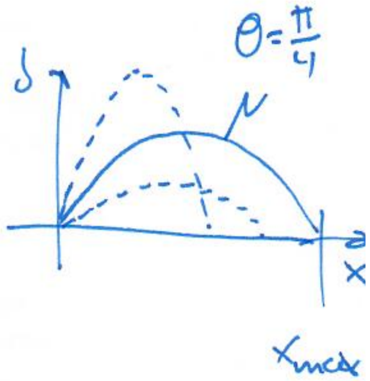
$$x(\theta) = \frac{v_0^2}{g}\sin\theta = K\sin(2\theta)$$

Maximum by deriving

$$x_{max} = 2K\cos(2\theta)\cdot\theta = 0$$

$$\cos(2\theta) = 0$$

$$\boxed{\theta = \frac{\pi}{4} = 45^\circ}$$



Maximum height

Condition $\boxed{v_y = 0}$

$$0 = v_y = v_{0y} - gt = v_0\sin\theta - gt$$

$$t = \frac{v_0\sin\theta}{g}$$

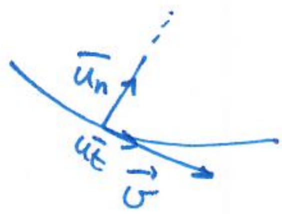
$$h = v_0\sin\theta\, t - \frac{1}{2}gt^2$$

$$\boxed{h = \frac{v_0^2\sin^2\theta}{2g}}$$

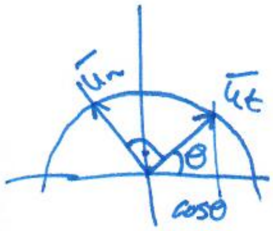
Intrinsic coordinates of acceleration

$$\bar{a} = \frac{d\bar{v}}{dt}$$

$$\bar{a} = \frac{d}{dt}(v\,\bar{u}_t) = \frac{dv}{dt}\,\bar{u}_t + v(t)\,\frac{d\bar{u}_t}{dt}$$



$$\bar{u}_t = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$



$$\bar{u}_t = (\cos\theta, \sin\theta)$$

$$\frac{d\bar{u}_t}{dt} = (-\sin\theta, \cos\theta)\frac{d\theta}{dt}$$

$$\frac{d\bar{u}_t}{dt} = \frac{d\theta}{dt}\,\bar{u}_n = \dot{\theta}\,\vec{u}_n$$

$$\dot{\theta} = \omega = \frac{v}{R} \Leftarrow \theta = l/R$$

$$\dot{\theta} = \dot{l}/R = v/R$$

$$\bar{a} = \bar{a}_t + \bar{a}_n = a_t\,\bar{u}_t + \frac{v^2}{R}\,\vec{u}_n$$

$\downarrow$

'normal'

'centripetal'

$R \equiv$ radius of curvature

curvature $= \frac{1}{R}$

Circular motion

$$\begin{cases} R = \text{constant} \\ a_t = 0 \\ a_n \text{ whatever} \end{cases}$$

if a_n constant, uniform circular

$$|v| = \text{constant} = \frac{\omega}{R}$$

$$a_n \text{ constant} = \frac{v^2}{R} = \omega^2 R$$

$$\omega = \frac{2\pi}{T} = 2\pi\nu \, ; \qquad v = \frac{2\pi R}{T} \quad \frac{[\text{length}]}{[\text{time}]}$$

If $\alpha \equiv \frac{d\omega}{dt}$ then $\alpha = \frac{d\dot{\theta}}{dt} = \ddot{\theta}$

$$\alpha = \frac{\omega_f - \omega_0}{t}$$

$$\boxed{\omega_f = \omega_0 + \alpha t}$$